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On turbulent boundary-layer separation

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An experimental and analytical study of the separation of a turbulent boundary layer is reported. The turbulent boundary-layer separation model proposed by Sandborn & Kline (1961) is demonstrated to predict the experimental results. Two distinct turbulent separation regions, an intermittent and a steady separation, with correspondingly different velocity distributions are confirmed. The true zero wall shear stress turbulent separation point is determined by electronic means. The associated mean velocity profile is shown to belong to the same family of profiles as found for laminar separation. The velocity distribution at the point of reattachment of a turbulent boundary layer behind a step is also shown to belong to the laminar separation family.

Prediction of the location of steady turbulent boundary-layer separation is made using the technique employed by Stratford (1959) for intermittent separation.

1. Introduction

Turbulent boundary-layer separation is normally listed as one of the most important unsolved problems in fluid mechanics. Much of the difficulty in treating turbulent boundary-layer separation stems from a carry over of laminar boundary-layer concepts. As will be demonstrated in the present paper, the laminar concepts are only a limited part of the turbulent separation picture. The picture of turbulent separation was greatly clarified when Kline (1957) demonstrated that turbulent separation is a spectrum of states. The first onset of separation is extremely unsteady with a reasonably steady separation observed downstream. The classical picture of separation implies that flow separates steadily along some appreciable length of line on a surface. This picture further requires that the derivative $(\partial U/\partial y)_{y=0}$ vanish continuously in both time and space along the line of separation. Kline's observation of steady separation appears to fit this classic model. Upstream of the steady separation region 'intermittent streaks of backflow' can be observed near the surface. It is possible to produce flows where the major portion of the boundary layer is affected by the intermittent separation.

The region of intermittent separation, as demonstrated by Sandborn & Kline (1961), proves to be the region identified as turbulent separation by most experimenters. Turbulent separation is experimentally identified either as the region where the transfer of mass decreases rapidly, or as the forwardmost penetration of a tracer. In either case, the region of intermittent separation rather than steady separation is identified. Although the mean wall shear stress in the intermittent separation region decreases rapidly, it cannot be zero until the fully separated region is reached.

Sandborn & Kline proposed a model for turbulent boundary-layer separation, which takes into account both the intermittent and fully developed turbulent separation regions. This model postulated that the mean velocity distribution at fully developed turbulent separation would be nearly identical to that of an equivalent laminar boundary-layer separation profile. This proposed model could not be experimentally verified, since no fully developed turbulent boundary-layer separation profile could be found in the literature.

The present paper demonstrates that the postulates regarding fully developed turbulent boundary-layer separation made by Sandborn & Kline can be verified experimentally. The velocity distribution of a turbulent boundary layer at the point of reattachment is also shown to fit the postulated separation model.

2. Experimental verification of the separation model

The model for turbulent separation proposed by Sandborn & Kline was the result of visual and empirical analysis of experimental measurements. It was convenient to view the whole separation region as a transition region from boundary layer to separated flow. A very general definition of this transition region might be the region where wall viscous effects are no longer important. Thus, by extending the concept of Clauser (1954) that the outer regions of turbulent boundary layer velocity profiles are laminar like in form, it was postulated that in the separation region the laminar characteristics should extend to the surface. The intermittent separation velocity profile would be equivalent to an adverse pressure gradient laminar boundary layer, which still has a finite shear at the surface. The fully developed turbulent separation profile would be equivalent to a laminar separation profile.

Obviously, the point of intermittent separation is not well defined in the model. A theoretical evaluation of a velocity profile at intermittent separation is not available. Thus, empirical results have been employed to correlate the velocity distributions at intermittent separation. For the present discussion it is adequate to note that the relation between form factor, H, and the ratio of displacement thickness, δ^*, to boundary-layer thickness, δ, for intermittent separation is given by Sandborn (1959)

$$H = 1+(1-\delta^*/\delta)^{-1}. \qquad (1)$$

Equation (1) was shown by Sandborn & Kline to correlate all 'turbulent' separation velocity profiles available. The minimum value of form factor at intermittent separation given by (1) is $H = 2$. Although there is mention in the literature of form factors as low as 1·8 at 'turbulent' separation, no such profiles could be located.

For fully developed (zero wall shear stress) turbulent separation, the velocity distribution is postulated to be equivalent to a corresponding laminar separation profile. A relation between H and δ^*/δ for laminar separation can be obtained from numerical solutions to the equations of motion, such as given by Liu & Sandborn (1967*a*). If the postulate is reasonable the fully developed turbulent separation velocity profiles must fall on the laminar separation correlation curve. Figure 1 is a plot of H versus δ^*/δ for both the intermittent separation and laminar (zero wall shear stress) separation correlations. The laminar correlation curve shown on figure 1 is from an empirical relation given by Sandborn (1959). The more exact numerical solutions agree with the empirical relation within the uncertainty in the value of the boundary-layer thickness.

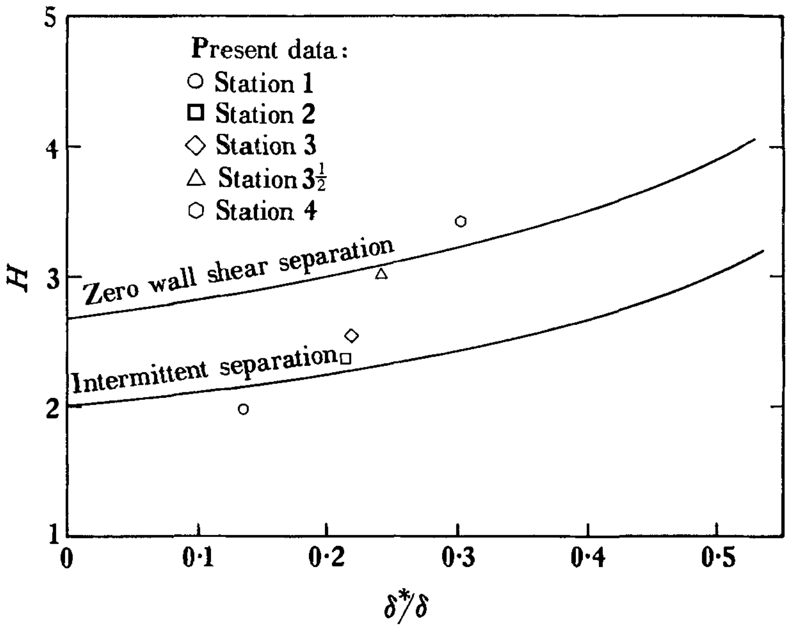

FIGURE 1. Separation correlations.

In order to check the postulate for the fully developed, zero wall shear stress, turbulent boundary-layer separation, a simple diffuser shaped duct was built into the CSU 6 × 6 ft. low-speed wind tunnel. Figure 2 (plate 1) is a schematic diagram of the duct. The free-stream velocity distribution along the duct is given in the table. The free-stream velocity at the minimum static pressure point of the flow is $U_1 = 34$ ft/sec.

The point of mean zero wall shear stress was determined by two independent methods. A special dual pressure probe was employed, with one probe pointing upstream and one pointing downstream. This pressure probe was traversed along the curved surface of the duct until the minimum differential pressure point was located. The minimum point was found to be at roughly $x = 50$ in., and was assumed to be the point of fully developed turbulent boundary-layer separation. A special hot-wire anemometer was also employed to determine the point where reverse flow occurred 50 % of the time. This system was employed by Plate & Lin (1965) to study reattachment flows. The system is similar to a technique outlined by Moon (1962). Two hot wires are mounted directly behind one another on a single probe. The two wires are very close together, so that one will always be in the wake of the other when the flow reverses. An electric circuit is employed to determine the percentage of time one wire is in the wake of the other. The point, where the measurements indicate that the flow (approaching $y = 0$) is reversed 50 % of the time, was assumed to be the point of fully developed separation. The magnitude of the flow in both directions must be equal in order that the assumption be valid. Figure 3 is a typical plot of the variation of percentage of time reverse flow with x distance for the hot-wire operating at several distances above the surface. As can be seen on figure 3, the 50 % location at $y = 0$ occurs at $x = 50$ in., which is in good agreement with the pressure probe results.

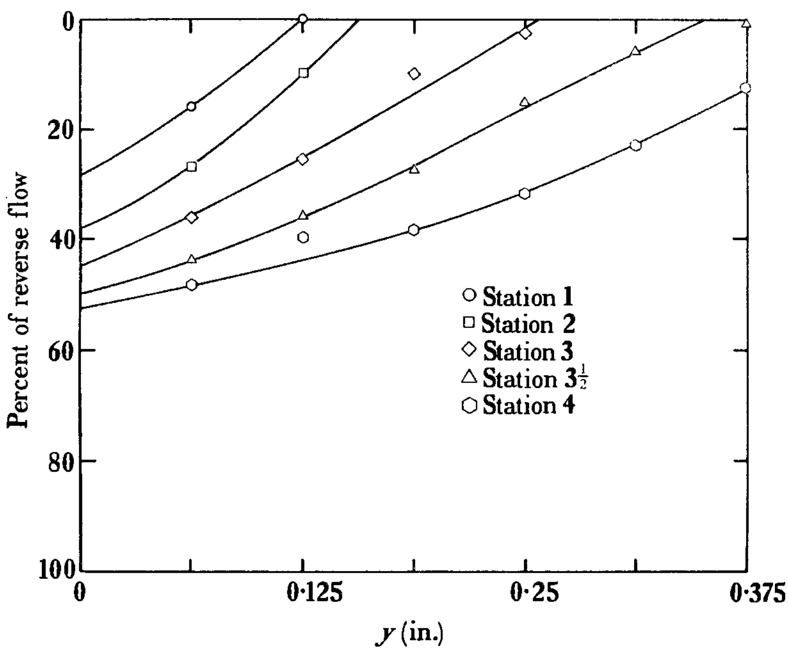

FIGURE 3. Percentage of flow reversal in the separation region.

An evaporation flow visualization technique was also examined to indicate the region of apparent turbulent separation. A definite change in evaporation rate could be observed near $x = 49$ in. while no change was observed in the area of $x = 50$ in. This visualization observation is usually reported for the turbulent separation location. It points out the ambiguity in the conclusions drawn from observations of 'turbulent' separation.

Velocity profiles measured with a pitot-static probe along the curved wall of the duct are shown in figure 4. The measurements have been corrected for the effect of turbulence and the static pressure variation through the boundary layer by the technique suggested by Landwebber (1960). Values of form factor, H and δ^*/δ obtained from the distributions shown in figure 4 are plotted on figure 1. Note that the value of H at station 2 agrees approximately with the intermittent separation correlation. Thus, the evaporation technique was indicating this intermittent separation region. However, of major interest is the fact that the value of H at $x = 50$ in. is very nearly that for the laminar separation correlation.

This agreement of fully developed, turbulent boundary-layer separation, velocity profile, form factor with the laminar correlation confirms the model proposed by Sandborn & Kline. Figure 5 compares this velocity profile at $x = 50$ in. with

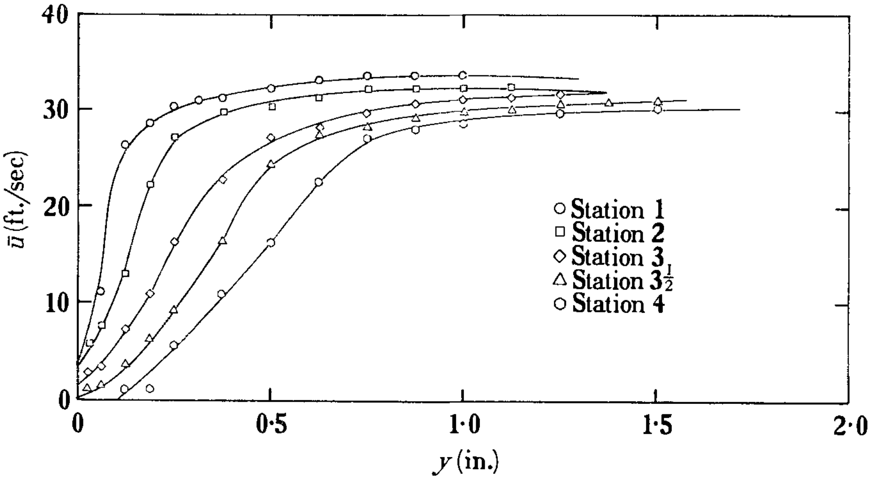

FIGURE 4. Measured mean velocity profiles in the separation region.

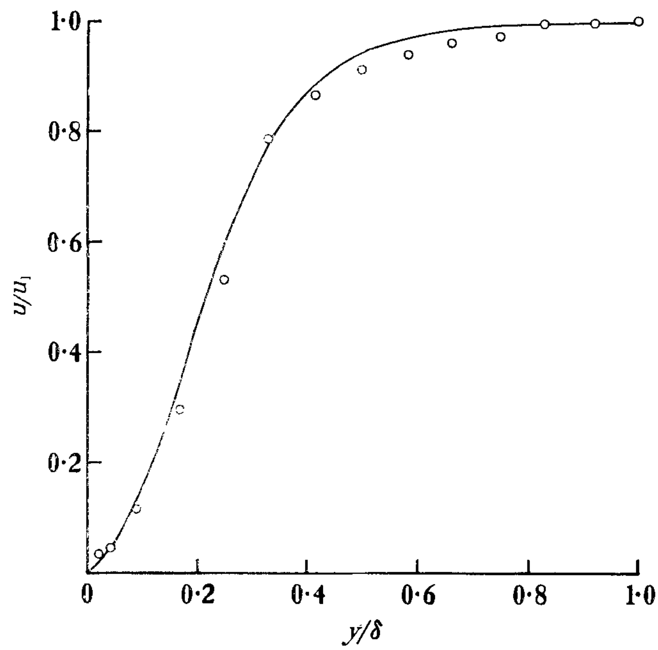

FIGURE 5. Comparison of the turbulent separation profile with a theoretical laminar separation profile.

a laminar boundary-layer separation profile of approximately the same form factor. The agreement between the measured fully turbulent separation profile and the equivalent laminar separation profile is further justification for the postulated model.

Turbulence measurements were made along the duct wall with the Colorado

State University constant temperature hot-wire anemometer described by Finn & Sandborn (1967). Figures 6*a* and *b* are plots of the longitudinal turbulent intensity and the turbulent shear stress across the boundary layer. These measurements can at best be taken as first-order approximations. The longitudinal

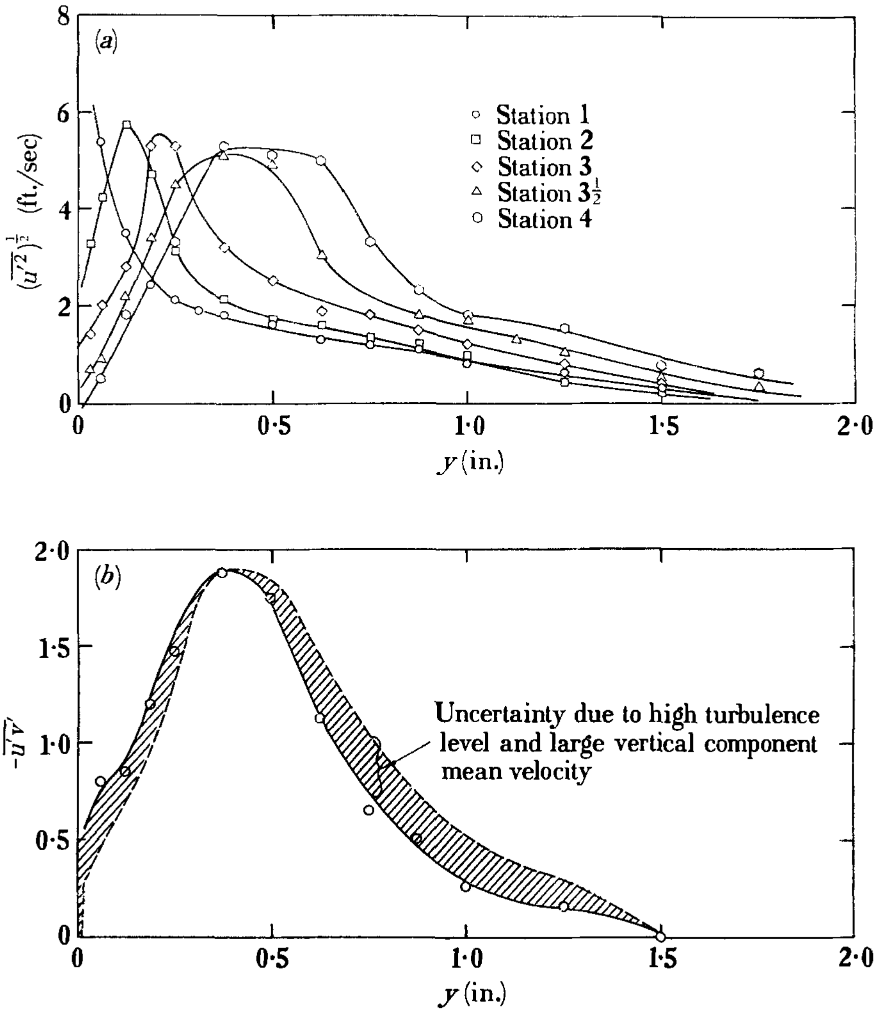

FIGURE 6. Turbulence measurements in the separation region. (*a*) Longitudinal velocity component. (*b*) Turbulent shear stress station.

velocity components were measured with a hot-wire normal to the mean flow and parallel to the surface. Direct graphic reading of the calibration curve for the hot wire eliminates the non-linear effect (Sandborn 1967). The non-linear calibration curve would have produced at most an error of 10% in the value of $(\overline{u^2})^{\frac{1}{2}}$. The major uncertainty is in the effect of the normal velocity component, v, on the hot-wire output. In the separation region very near the surface the mean velocity is nearly zero, and the total velocity is approximately

$$V_{tot} = (u^2+v^2)^{\frac{1}{2}}.$$

Thus, the heat loss is equally affected by both the u and v fluctuations. As an upper limit on the error in assuming the wire normal to the mean flow measures $(\overline{u^2})^{\frac{1}{2}}$, consider the case where $u = v$ and the mean flow is zero. This upper limit indicates an error of 41 % in the indicated value of $(\overline{u^2})^{\frac{1}{2}}$. This maximum error could occur only in the $(y \rightarrow 0)$ region near the wall. However, since the boundary conditions on v are more restrictive than on u near the surface, it is expected that

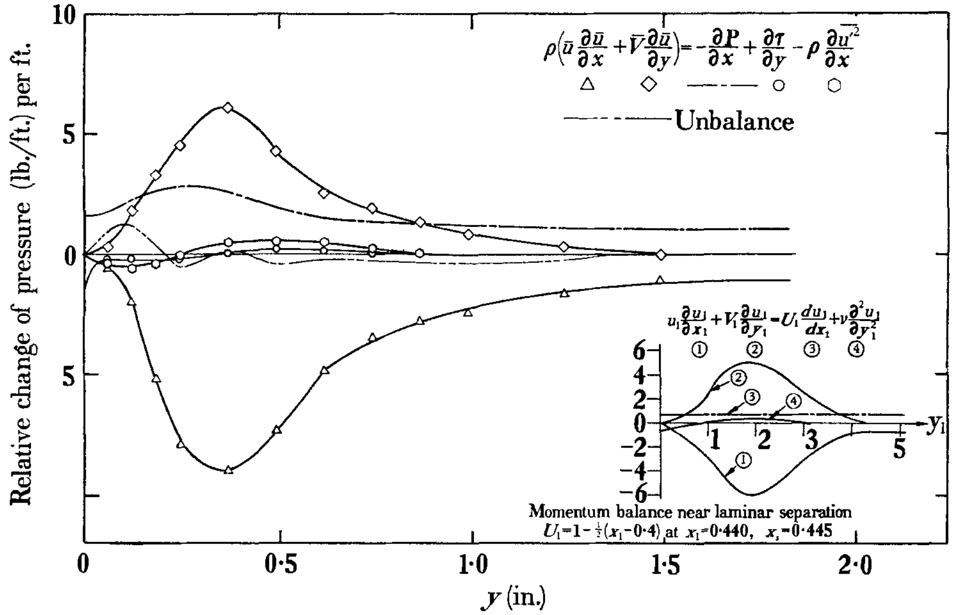

FIGURE 7. Balance of the x-direction momentum equation in the separation region.

the error in u is much less than 41 %. A similar analysis leads to the same possible uncertainty in $\overline{uv}$ near the surface. The measurements of $\overline{uv}$ were made with x-wire probes according to the technique outlined by Sandborn (1967). An error in the $\overline{uv}$ measurements is also encountered in the outer part of the boundary layer due to the large vertical mean velocity component. Figure 6*b* shows the actual measured points without corrections. The dashed curve on figure 6*b* is the estimated correct values of $\overline{uv}$, taking into account the corrections noted above. The correction near the wall is the maximum possible, so it is thought the actual value of $\overline{uv}$ lies within the shaded area of figure 6*b*.

From the measured velocity distribution, static pressure distribution and the turbulent quantities, an approximate balance of the x-direction momentum equation was made. Figure 7 shows the variation of the terms in the equation as a function of y for $x = 49{\cdot}75$ in. The insert on figure 7 shows a typical laminar boundary-layer x-direction momentum equation balance obtained from the numerical solutions of Liu & Sandborn (1967*b*). The comparison of the laminar and turbulent x-momentum equations explains in part why the velocity profiles are nearly identical. In both the laminar and the turbulent case the inertia terms dominate the equation. The assumption of a constant eddy viscosity to represent $\overline{uv}$, as required by Clauser's outer region similarity analysis, is obviously of little importance in the separation region. With perhaps the exception of the region very near the wall, the x-momentum equations for both laminar and turbulent separation are similar, with only the inertia and pressure gradient terms of importance.

3. Other zero wall shear stress turbulent boundary layers

Since no independent checks of the turbulent-laminar similarity at separation are known, other types of zero wall shear stress turbulent boundary layers have been considered. A typical zero wall shear profile is that of a reattaching turbulent shear flow. It is not obvious that reattachment is equivalent to separation, however it is found experimentally that the velocity distributions for the two cases are similar. Figure 8 shows several typical reattachment profiles compared with laminar separation profiles. The data of Plate & Lin (1965) were also determined by the hot-wire reverse flow technique used in the present experimental measurements. In all cases the experimental data agree well with the laminar separation profiles. The laminar separation profiles have approximately the same values of form factor as the measurements; however, exact matching of form factors was not feasible.

4. Separation prediction

As shown on figure 7, the boundary-layer approximation to the equations of motion is valid for the turbulent boundary layer near separation.

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 U}{\partial y^2}-\frac{\partial \overline{uv}}{\partial y}, \tag{2}$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+\frac{\partial \overline{v^2}}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}. \tag{3}$$

It appears that the turbulent shear stress term $\partial\overline{uv}/\partial y$ can also be neglected, except very near the wall. Turbulent terms, such as $(\partial\overline{u^2}/\partial x)$ are also found to be of minor importance for the balance shown in figure 7. No other term could be found in the complete equation of motion that could affect the results of figure 7. The unbalance near the wall for the experimental data shown in figure 7 occurs in the region where measurement uncertainties are the greatest. The y-direction equation of motion has not been evaluated, so it is possible that the inertia terms are not needed. It is important to note that the variation of static pressure with y-distance has a marked effect on the balance of the x-direction equation.

Near the surface the usual turbulent boundary-layer assumption is that the inertia terms can be neglected compared to $\nu(\partial^2 U/\partial y^2)$ and $\partial\overline{uv}/\partial y$. Thus

$$\nu\frac{\partial^2 U}{\partial y^2}-\frac{\partial \overline{uv}}{\partial y}\approx\frac{1}{\rho}\frac{\partial p}{\partial x}\ (\text{for } y \text{ small}). \tag{4}$$

For the limited region of very small y, where (4) is a reasonable approximation, $\partial p/\partial x$ is assumed to be independent of y. Integrating (4) gives

$$\nu\frac{\partial U}{\partial y}-\overline{uv}=\frac{y}{\rho}\frac{\partial p}{\partial x}+C. \tag{5}$$

For fully developed turbulent separation the constant of integration will be zero, since all the terms are zero at $y = 0$. For intermittent separation and non-separated flow, $C = \tau_w/\rho$. Stratford (1959) considered equation (5) for the 'separation' case; $C = 0$ and $\overline{uv} \gg \nu\partial U/\partial y$. The turbulent shear stress is expressed by Prandtl's mixing length assumption

$$-\rho\overline{uv} = k^2 l^2 \left|\frac{\partial U}{\partial y}\right| \frac{\partial U}{\partial y} = k^2 y^2 \left(\frac{\partial U}{\partial y}\right)^2 \quad \text{for} \quad \frac{\partial U}{\partial y} > 0, \tag{6}$$

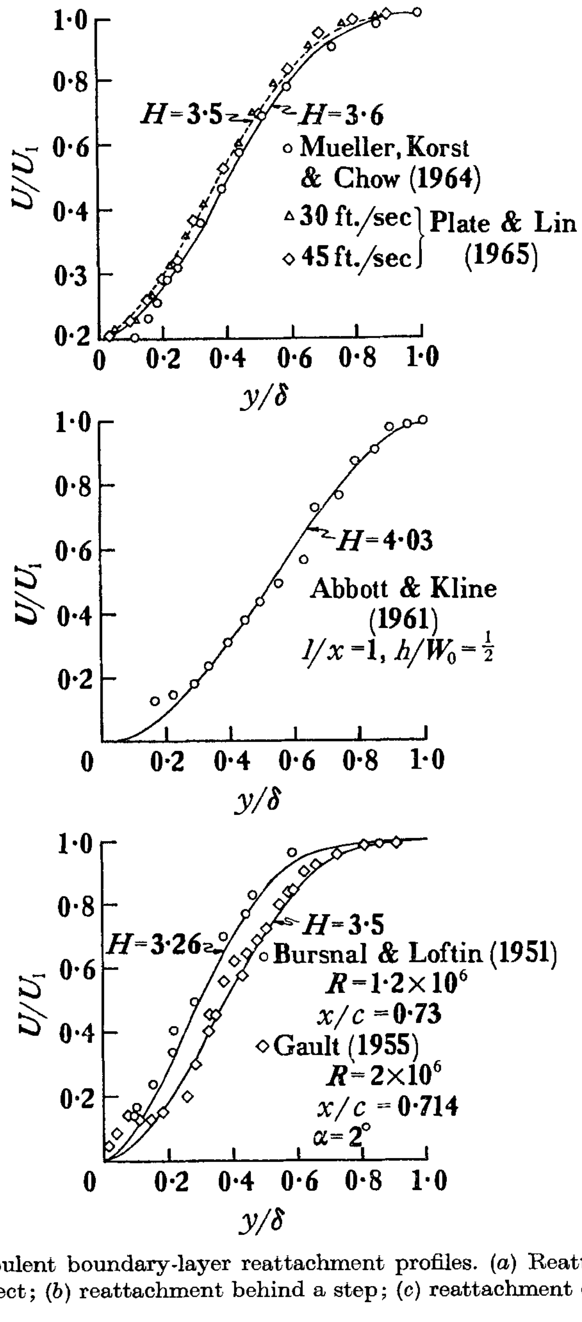

FIGURE 8. Turbulent boundary-layer reattachment profiles. (*a*) Reattachment behind a blunt object; (*b*) reattachment behind a step; (*c*) reattachment on an airfoil.

so that a solution for the mean velocity distribution is obtained in the form

$$U = \left(\frac{4y}{k^2\rho}\frac{\partial p}{\partial x}\right)^{\frac{1}{2}}. \tag{7}$$

Stratford suggests that (7) may be regarded as the first term of a series expansion representing the whole inner part of the velocity profile. However, it is difficult to accept this suggestion, since the value of $(\partial U/\partial y)_{y=0}$ would be infinite rather than zero.

In order to represent the fully developed turbulent separation velocity profile near the surface, a relation other than (6) must be employed. In order to obtain a zero value of the wall shear, the mixing length must vary as a power of y less than $\frac{1}{2}$ (i.e. $l^2 = Ly$ produces $U = y(\rho^{-1}\partial p/\partial x)^{\frac{1}{2}}$ which gives a finite value for the wall shear). Another possibility is to assume an eddy viscosity relation, rather than a mixing length. The concept of an eddy viscosity would be in keeping with the concepts of Clauser extended to the separation region.

$$-\rho\overline{uv} = \epsilon_\tau(\partial U/\partial y), \tag{8}$$

using (8) in (5) and solving for U yields ($\tau_w = 0$ and ϵ_τ = constant)

$$U = \frac{y^2}{2}\frac{\partial p}{\partial x}\left(\frac{1}{\mu+\epsilon_\tau}\right). \tag{9}$$

Equation (9) gives a zero slope for the mean velocity gradient at the surface, as required by $\tau_w = 0$. Although it is possible to arrive at a laminar-turbulent equivalence from an examination of the terms in the equation of motion, the original postulate of Sandborn & Kline was based on an extension of Clauser's (1954) outer region similarity analysis. If it is assumed that the outer region extends to the wall then (8) is a statement of this assumption.

The assumption of constant eddy viscosity is an approximation even for the outer region of zero pressure gradient turbulent boundary layers, as demonstrated by Rotta (1962, 1965). However, as pointed out by Rotta, the variation of ϵ_τ has only a second-order effect on the velocity distribution. Away from the separation region where $\tau_w > 0$, the solution of (5) very near the surface, is

$$U = \frac{y^2}{2\mu}\frac{\partial p}{\partial x} + \frac{y\tau_w}{\mu} \quad (\text{for } \overline{uv} \to 0). \tag{10}$$

Thus, (9) is a reasonable approximation in the region of the surface at separation. Intermittent separation might also be included by replacing μ by $\mu+\epsilon_\tau$ in (10). However, the problem of intermittent separation will be in the selection of a reasonable value for τ_w.

Liu (1967) has considered the prediction of the location of fully developed separation following the analytical technique employed by Stratford (1959). Using an upstream power law velocity profile (i.e. $U'/U_0 = (y'/\delta')^{1/n}$) and equation (9) neglecting μ, the location of separation, x_s, is obtained from the relation

$$C_p^{\frac{1}{2}(2n-1)} x_5 \frac{dC_p}{dx} = C_1 \frac{R^{\frac{2}{5}}}{Re_\tau}, \tag{11}$$

where $\quad C_p = \dfrac{p-p_0}{\frac{1}{2}\rho U_0^2}, \quad R = \dfrac{\rho U_0 x}{\mu}, \quad R_\tau = \dfrac{\rho U_0 x}{\epsilon_\tau}$

and
$$C_1 = \frac{\left(\frac{2(n+1)}{3}\right)^{\frac{1}{2}} \left(\frac{2n+2}{2n-1}\right)^{\frac{1}{2}(1-2n)}}{0{\cdot}036(n+1)^2(n+2)^2}.$$

As might be expected, the turbulence expressed by the eddy viscosity has an effect on the location of separation. For large-scale turbulence the eddy viscosity will be large and (11) will indicate a greater distance x_s to separation. This result

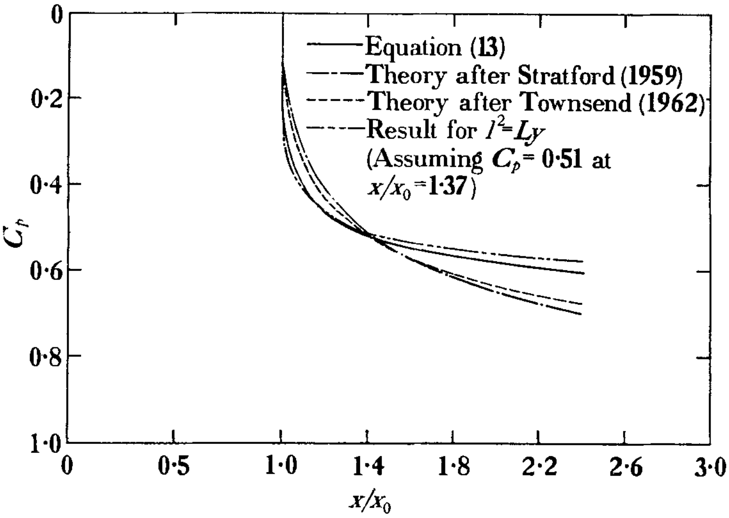

FIGURE 9. Pressure distribution for flow with zero wall shear stress.

appears completely consistent with the general concepts of turbulence effects. In order to allow an engineering approximation of (11) it was estimated that $(R_\delta^{\frac{2}{3}}/R_\tau)^{\frac{1}{2}} = 0{\cdot}22$ for the present flow studied. Thus,

$$C_p^{\frac{1}{4}(2n-1)} \left(x_s \frac{dC_p}{dx}\right)^{\frac{1}{2}} \simeq 0{\cdot}22 C_2 \quad (\text{for } C_p \leqslant 0{\cdot}5), \tag{12}$$

C_2 varies from 0·377 for $n = 6$ to 0·24 for $n = 8$.

Following Stratford's (1959) technique for the calculation of a pressure gradient that produces a continuously zero wall shear stress velocity distribution, (12) can be integrated to give

$$C_p = \left[0{\cdot}0484 C_1^2 (n+\tfrac{1}{2}) \ln \frac{x}{x_0}\right]^{2/(2n+1)} \tag{13}$$

Equation (13) is compared with the calculations obtained by Stratford (1959) and Townsend (1962) in figure 9. As might be expected, the initial gradient is somewhat steeper for the fully developed separation profile given by (9), compared with the results of Stratford and Townsend which employed equation (7). For comparison the relation obtained from the linear mixing length assumption ($l^2 = Ly$) is also shown in figure 9. The true zero wall shear stress velocity profile is not likely to be as stable as the infinite wall shear stress profile of Stratford. Thus, the final pressure gradient that can be withstood by the zero wall shear profile is less than that of Stratford's profiles.

5. Concluding remarks

The present experimental study confirms the proposed model that laminar and turbulent separation with zero wall shear stress are similar. The intermittent separation region was also postulated by Sandborn & Kline to be laminar-like in character. It was suggested from isolated experimental evidence that the velocity profiles at intermittent separation were equivalent to unsteady laminar separation velocity profiles. Sandborn & Kline found that the intermittent turbulent separation correlation corresponded to laminar-turbulent transition boundary-layer profiles. Early in the research on laminar-turbulent transition it was suggested that flow separation might play a part. Since measured velocity distributions failed to indicate the classical zero wall shear stress profile, separation was not considered as a part of laminar-turbulent transition. One may now wonder if unsteady separation is perhaps a necessary part of laminar-turbulent transition. The fact that Schubauer & Scramstead (1948) reported the observation of reverse flow in the transition region is a strong indicator that unsteady separation does indeed play a part.

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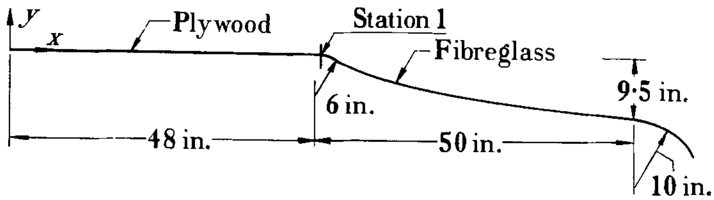

Top view

Station	x Distance (in.)	U_1 (ft./sec)	δ^* (in.)	θ (in.)
1	48·75	33·5	0·103	0·0526
2	49·25	32·2	0·162	0·0687
3	49·75	31·7	0·275	0·108
$3\frac{1}{2}$	50·00	31·2	0·363	0·121
4	50·25	30·7	0·455	0·133

FIGURE 2. Separation flow model.